$$F'(y+z)-F'(y)=0.$$

Then $F'(y)$ must be a constant and $F(y)$ will have the form

$$F(y)=ay+b.$$

Also $f(x)$ and $\theta(x)$ will have the relations, respectively:

$$f(2x)=af(x)+b, \quad \theta(2x)=a\theta(x).$$

Therefore both $f(x)$ and $\theta(x)$ must be constants. Hence we have the following theorem:

The uniform analytical function satisfying a given addition theorem is the only one which satisfies the equation acquired by making $x=y$ on the addition theorem.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

The following note will be of value, since it calls attention to the fact that unless the law of a series is given, the *definite* determination of the series is impossible. If only a certain number of terms of a series is given and not the law of the series, the actual law of the series can in no case be really determined. All that can be done is to find the *simplest law* which the few given terms will obey. The solutions referred to may be justified on the ground that they were attempts to find this *simplest law*. Ed. F.

NOTE ON THE SOLUTION OF PROBLEM 266, IN AUGUST-SEPTEMBER MONTHLY.

By CLARENCE E. COMSTOCK, Bradley Polytechnic Institute.

Find the nth term and the sum of n terms of the series $1+3+7+17+\ldots$

One writer assumes without the least excuse for such an assumption that it is an arithmetical series of the third order and proceeds by the method of finite differences, getting a result in accord with his assumption. Five terms of his series are $1+3+7+17+37+\ldots$

Another writer assumes with a little more plausible excuse that it is a recurring series of the second order and, of course, gets a result in accord with his assumption. Five terms of his series are $1+3+7+17+41+\ldots$

Suppose I make the assumption that it is an arithmetical series of the fourth order. I can then build up

$$1+3+7+17+38+76+\ldots \text{ or } 1+3+7+17+39+81+\ldots$$

or as many more as I care to take the time to construct, on the one supposition that it is an arithmetical series of the fourth order. Or let me assume it a recurring series of the fourth order, and get, using the scale of relation,

$$u_n=2u_{n-1}+u_{n-2}+0u_{n-3}+u_{n-4}, \text{ say, } 1+3+7+17+42+104+\ldots$$

The fact is, the problem is absolutely without meaning. There is nothing in the sequence $1+3+7+17+\ldots$ to show what the next term is. A series is not determined until its law is in some way stated or indicated.

267 Proposed by O. E. GLENN, Ph. D., Philadelphia, Pa.

Express the trigonometric functions of x as infinite continued fractions.

Solution by R. D. CARMICHAEL, Professor of Mathematics, Anniston, Ala.; and J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Let the series $u=A-B+C-D\ldots=\cfrac{a_1}{c_1+\cfrac{a_2}{c_2+\cfrac{a_3}{c_3+\ldots}}}$

To determine the a's we have the following (Euler's "Introductio," I, §367):

$$a_1=A.c_1,\ a_2=\frac{B.c_1c_2}{A-B},\ a_3=\frac{AC.c_2c_3}{(A-B)\,(B-C)},\ a_4=\frac{BD.c_3c_4}{(B-C)\,(C-D)}\cdots$$

in which the c's are to be so chosen that the a's are integral functions. Applying these formulas, we have

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots=\cfrac{x}{1+\cfrac{x^2}{2.3-x^2+\cfrac{2.3x^2}{4.5-x^2+\ldots}}}$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots=\cfrac{1}{1+\cfrac{x^2}{1.2-x^2+\cfrac{1.2x^2}{3.4-x^2+\cfrac{3.4x^2}{5.6-x^2+\ldots}}}}$$

$$\tan x=\frac{2}{\pi-2x}-\frac{2}{\pi+2x}+\frac{2}{3\pi-2x}-\frac{2}{3\pi+2x}+\frac{2}{5\pi-2x}+\ldots^*$$

$$=\cfrac{2}{\pi-2x+\cfrac{4x}{(\pi-2x)^2+\cfrac{2\pi}{\pi^4-4x^2+\cfrac{(\pi-2x)(3\pi-2x)^2\cdot}{2x(\pi+2x)+\ldots}}}}$$

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*$Encyclopedia\ Britannica$, Vol. XXIII, p. 572.